Weak Base in Water

For a solution of a weak base in water, the determination of all species concentrations follows analogously with the acid case; i.e., four equations in four unknowns to which simplifications can often be made.

$$B + H_2O \rightleftharpoons BH^+ + OH^-$$

$$2H_2O \rightleftharpoons H_3O^+ + OH^-$$

$$K_b = \frac{[BH^+][OH^-]}{[B]}$$

②
$$K_w = [H_3O^+][OH^-] = 1.00 \times 10^{-14}$$

The charge balance expression is

$$(OH^{-}] = [OH^{-}]_{base} + [OH^{-}]_{water} = [BH^{+}] + [H_{3}O^{+}]$$

The mass balance expression is

$$(B) = C_B - [BH^+]$$

Weak Base In Water Simplifying Assumptions

$$B + H_2O \rightleftharpoons BH^+ + OH^-$$

$$2H_2O \Rightarrow H_3O^+ + OH^-$$

Assumption I: If $C_{\rm B} >> 10^{-7}$ M, assume virtually all OH⁻ comes from B. Thus,

$$[OH^-] \approx [HB^+]$$

Assumption II: If $C_B >> K_b$ (usually 2 powers of 10 or more), assume B is negligibly hydrolyzed. Thus,

[B]
$$\approx C_{\rm B}$$

If both assumptions can be made:

$$K_b = \frac{[OH^-][BH^+]}{[B]} = \frac{[OH^-]^2}{C_B}$$

and

$$[OH^{-}] = \sqrt{C_B K_b}$$

Breakdown of Assumption II

If $C_B \approx K_b$ (usually within 2 powers of 10), assume B is *not* negligibly hydrolyzed. Thus,

$$[B] = C_{B} - [OH^{-}]$$

$$K_{b} = \frac{[OH^{-}][BH^{+}]}{[B]} = \frac{[OH^{-}]^{2}}{C_{B} - [OH^{-}]}$$

$$[OH^{-}]^{2} + K_{b} [OH^{-}] - K_{b}C_{B} = 0$$

Solve the quadratic equation for $[OH^-] = [BH^+]$

K_a 's of Polyprotic Acids

• We can define a K_a for each dissociation of a polyprotic acid.

$$H_3PO_4 + H_2O \rightleftharpoons H_3O^+ + H_2PO_4^- \qquad K_1 = 7.52 \times 10^{-3}$$
 $H_2PO_4^- + H_2O \rightleftharpoons H_3O^+ + HPO_4^{2-} \qquad K_2 = 6.23 \times 10^{-8}$
 $HPO_4^{2-} + H_2O \rightleftharpoons H_3O^+ + PO_4^{3-} \qquad K_3 = 4.5 \times 10^{-13}$

• As is typical of polyprotic acids, $K_1 > K_2 > K_3$, etc.

Determining [H₃O⁺] for a Polyprotic Acid

- All equilibria occur simultaneously, but K_1 is usually so much greater than K_2 and K_3 that in a moderately concentrated solution we can assume that virtually all of the hydronium ion comes from the first step.
- ✓ Charge balance expression:

$$[H_{3}O^{+}] = [H_{3}O^{+}]_{1} + [H_{3}O^{+}]_{2} + [H_{3}O^{+}]_{3} + [H_{3}O^{+}]_{water}$$

$$= [H_{2}PO_{4}^{-}] + [HPO_{4}^{2-}] + [PO_{4}^{3-}] + [OH^{-}]$$

$$\approx [H_{2}PO_{4}^{-}]$$

$$\approx Assumption I$$

- ✓ For calculations of a pure polyprotic acid in water (with no additional conjugate base) we only need to consider K_1 to find $[H_3O^+]$.
- For polyprotic acids in general, this assumption is valid only if the successive K's are widely separated; i.e., $K_1 >> K_2 >> K_3$, etc.

Simplifying Assumptions for a Polyprotic Acid, H,A

Assumption I: If $C >> 10^{-7}$ M, assume virtually all H_3O^+ comes from the first dissociation. Thus, for H_nA ,

$$[\mathbf{H}_3\mathbf{O}^+] \approx [\mathbf{H}_{(n-1)}\mathbf{A}^-]$$

Assumption II: If $C >> K_1$ (usually 2 powers of 10 or more), assume the acid is negligibly dissociated. Then, for H_nA ,

$$[H_nA] \approx C$$

Breakdown of Assumption II: If $C \approx K_1$ (usually within 2 powers of 10), assume the acid is *not* negligibly dissociated. Then, for H_nA ,

$$[H_nA] = C - [H_3O^+]$$

Calculation Procedure for a Polyprotic Acid, H_3A $K_1 >> K_2 >> K_3$

- ① Use K_1 and Assumption I with or without Assumption II to calculate $[H_3O^+] = [H_2A^-]$.
- ② $[HA^{2-}] \approx K_2$.
- ③ Use K_3 and the previously calculated values of $[H_3O^+]$ and $[HA^{2-}]$ to calculate $[A^{3-}]$.
- 4 Use K_w and the previously calculated value of $[H_3O^+]$ to calculate $[OH^-]$.
- (5) Check Assumption I by substituting calculated values into the exact charge balance expression to verify

$$[H_3O^+] = [H_2A^-] + [HA^{2-}] + [A^{3-}] + [OH^-] \approx [H_2A^-]$$

⑤ If using Assumption II, check to make sure the error in the mass balance expression is acceptably small; i.e.,

$$[H_3A] = C - [H_3O^+] \approx C$$

and
$$([H_2A^-]/C) \times 100\% < 10\%$$